A Link between Large- and Small-scale Intermittency in Stratified Turbulent Flows

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Large-scale Intermittency in Stratified Flows

A characteristic feature of turbulent flows, including geophysical flows, is the so-called internal (or small-scale) intermittency, producing localized intense variations of the energy dissipation and of field gradients, as observed in many instances in the atmosphere, the ocean, in laboratory experiments and in direct numerical simulations (DNSs). Internal intermittency is also at the origin of the non-Gaussian behavior of the probability distribution functions (PDFs) of small-scale turbulent velocity fluctuations.

Intermittent events at large scales have been observed in clear air turbulence with patches that can span up to 100 km horizontally and 1 km vertically [1], in the vertical velocity and temperature in atmospheric mesoscales [2], in the mesosphere-lower thermosphere ([3]), in the ocean (in observations [4] and models), and in DNSs [5,6] where large-scale bursts develop in a certain parameter space.

Indeed, by carrying out a parametric study on DNSs of the Boussinesq equations, we demonstrated that the vertical velocity w and the buoyancy θ (proportional to potential temperature fluctuations) are highly intermittent at large scales in DNSs of the Boussinesq equations in a range of parameters of geophysical interest [5,6]. The intermittent behaviour of w is associated with the presence of extreme events in the vertical component of the velocity field, emerging as powerful vertical "drafts" which affect the overall dynamics of the flow including its mixing properties [5].

Unstable regions of the flow, identified using the gradient Richardson number, $Ri = [N (N - \partial_z\theta) / (\partial_z \boldsymbol{u}_\perp)^2]$,
were also qualitatively linked to the extreme vertical drafts using 3D renderings of the fields [5], as shown in the figure below:

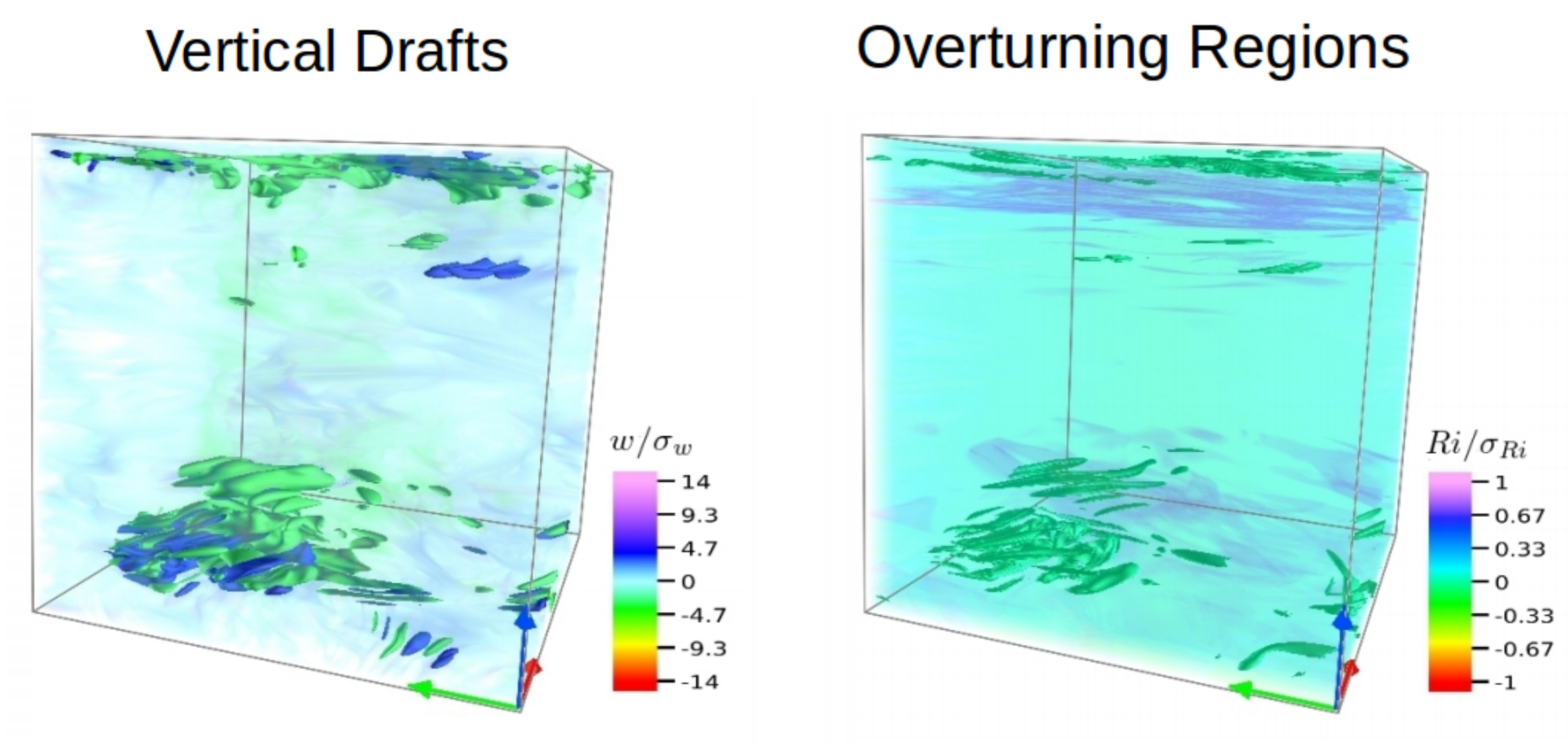

Fig. 1: Left: Rendering of w for for run 9 (Fr = 0.076, Tab. 1). A threshold is used to highlight the presence of intense vertical drafts ($w > 3\sigma_w$) which appear as large-scale structures emerging in the flow.
Right: Rendering of the gradient Richardson number where regions prone to develop overturning, with $Ri/\sigma_{Ri} < 0.004$, are visualized using opaque colors.

Numerical Framework and Parameters

We carried out 17 DNSs of the Navier-Stokes equations in the Boussinesq framework in presence of stable stratification. To ensure incompressibility, the velocity field $\boldsymbol{u}$ satisfies the condition $\nabla\cdot\boldsymbol{u}=0$.

The integrated dimensionless equations are:

$$\partial_t \boldsymbol{u}+(\boldsymbol{u}\cdot\nabla)\boldsymbol{u} \;=\; -\nabla p - N\theta\hat{z}+\boldsymbol{F}+\nu\nabla^2\boldsymbol{u}$$
$$\partial_t\theta + \boldsymbol{u}\cdot\nabla\theta \;=\; Nw+\kappa\nabla^2\theta$$

The initial conditions consist of vanishing buoyancy fluctuations θ = 0 and a velocity field with kinetic energy randomly distributed on spherical shells with wavenumbers k_F = [2,3].
A random isotropic mechanical forcing **F** is applied to the velocity field at the same wavenumbers. We performed 17 runs (see Tab. 1) on isotropic grid of 512^3 points, with the size of the periodic 3D box equal to 2π.
We integrated the equations using the Geophysical High-Order Suite for Turbulence [7], a hybrid MPI-, OpenMP- and CUDA-parallelized pseudo-spectral code.

The intermittency of $\boldsymbol{u}$=(u, v, w) and θ *is* characterized by their dimensionless fourth-order moment, the kurtosis,

$$K_\alpha \;=\; \frac{\langle(\alpha-\bar{\alpha})^4\rangle}{\langle(\alpha-\bar{\alpha})^2\rangle^2}$$

With α = u, v, w, θ or field gradients. The reference value is 3, which is associated to Gaussian distributions. Values of $K_\alpha > 3$, instead, corresponding to leptokurtic PDFs with fat tails and a higher probability of extreme values.

Run	$Re[\times10^3]$	Fr	R_B
1	3.9	0.015	0.87
2	3.8	0.026	2.5
3	3.8	0.030	3.4
4	3.8	0.038	5.6
5	3.8	0.044	7.3
6	3.8	0.051	10.2
7	3.9	0.068	17.7
8	3.8	0.072	19.7
9	3.8	0.076	22.1
10	3.8	0.081	25.2
11	3.7	0.098	35.9
12	3.6	0.11	47.5
13	3.0	0.16	75.2
14	2.6	0.19	90.9
15	2.6	0.28	201
16	2.8	0.56	895
17	2.9	0.93	2560

Tab. 1: Parameters of the 17 DNS performed in this study: the Reynolds number Re = UL/ν, the Froude number Fr = U/LN and the buoyancy Reynolds R_b=Re Fr^2.
U and L are the charateristic velocity and length scale of the flow respectively, N is the Brunt-Vaisala frequency and ν the kinematic viscosity. Every run has been performed with a resolution of 512^3 grid points. In red the simulations which develop large-scale extreme events.

Connecting Large- and Small-scale Intermittency

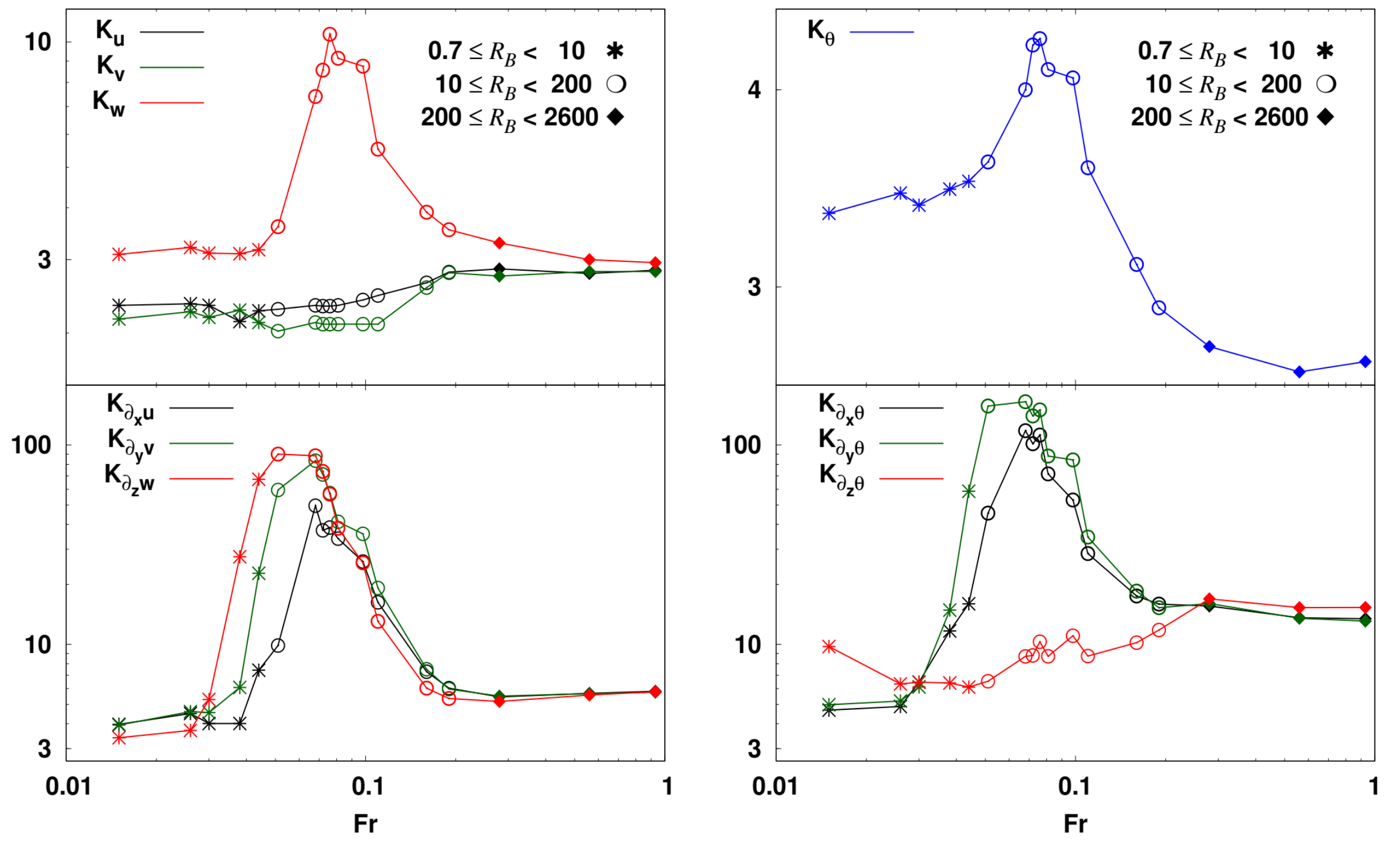

Fig. 2: Kurtosis of the large-scale (top*)* and small-scale fields (bottom) as a function of the Froude number Fr for all the 17 runs.

- We observe a clear correlation between the large-scale intermittent behavior of the vertical velocity (w) and the small-scale intermittency of its gradient components, detected through the kurtosis
- Even though the horizontal velocity components (u, v) show no evolution with Fr, the kurtosis of their gradients displays a trend with Fr following that of the large-scale intermittent behavior of the vertical component w
- A trend similar to K_w is also observed for K_θ and its gradients, with the exception of $\partial_z\theta$, probably due to stable stratification imposed in the Boussinesq model
- The overall behavior of the gradient components statistics suggests isotropization of the small scales.

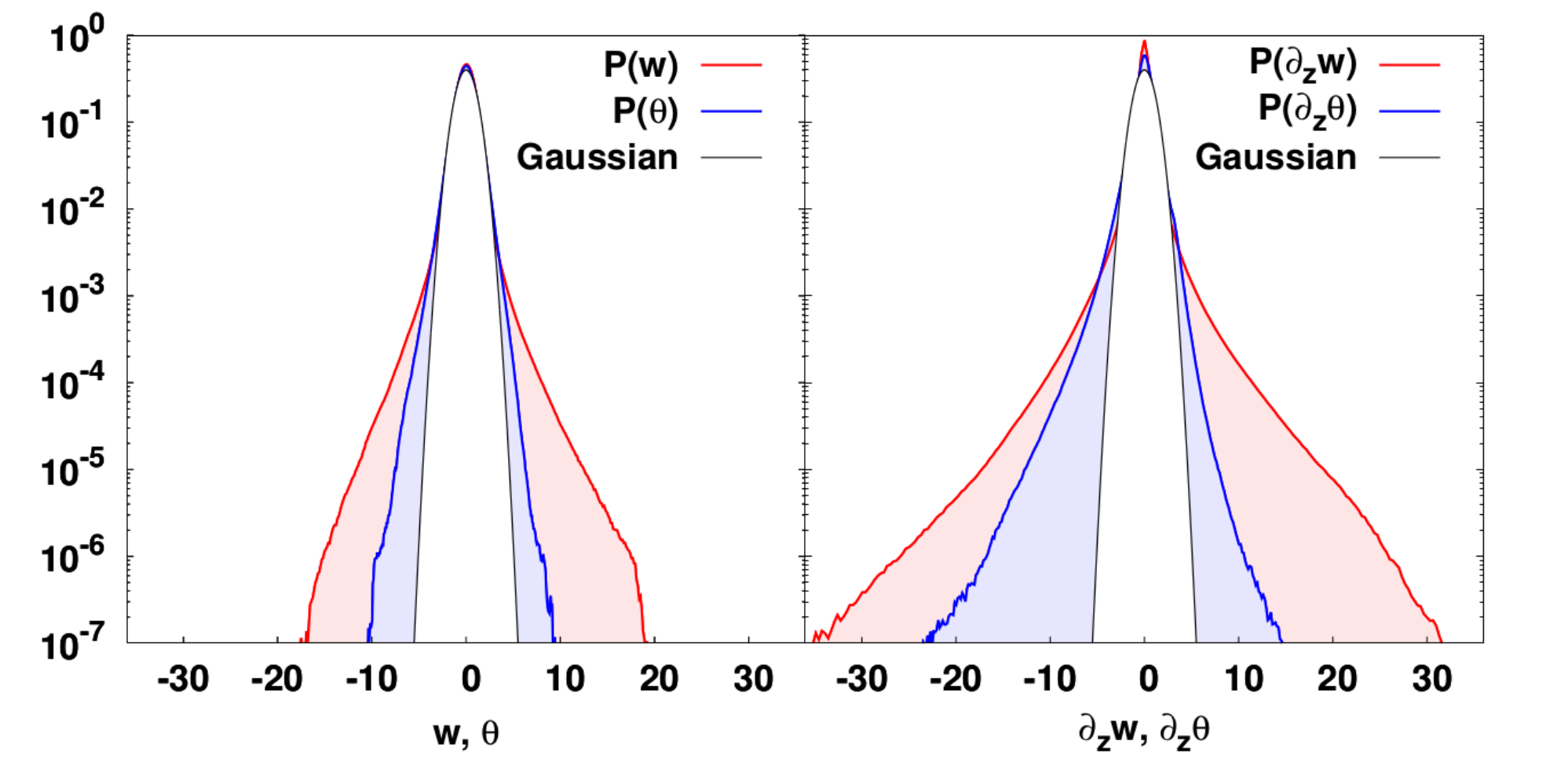

Fig. 3: Left: PDFs of the vertical velocity w (red) and the buoyancy θ (blue) for run 9 (Fr = 0.076, Tab. 1). Right: PDFs of their respective derivative with respect to z for the same run. The black line represents the Gaussian reference.

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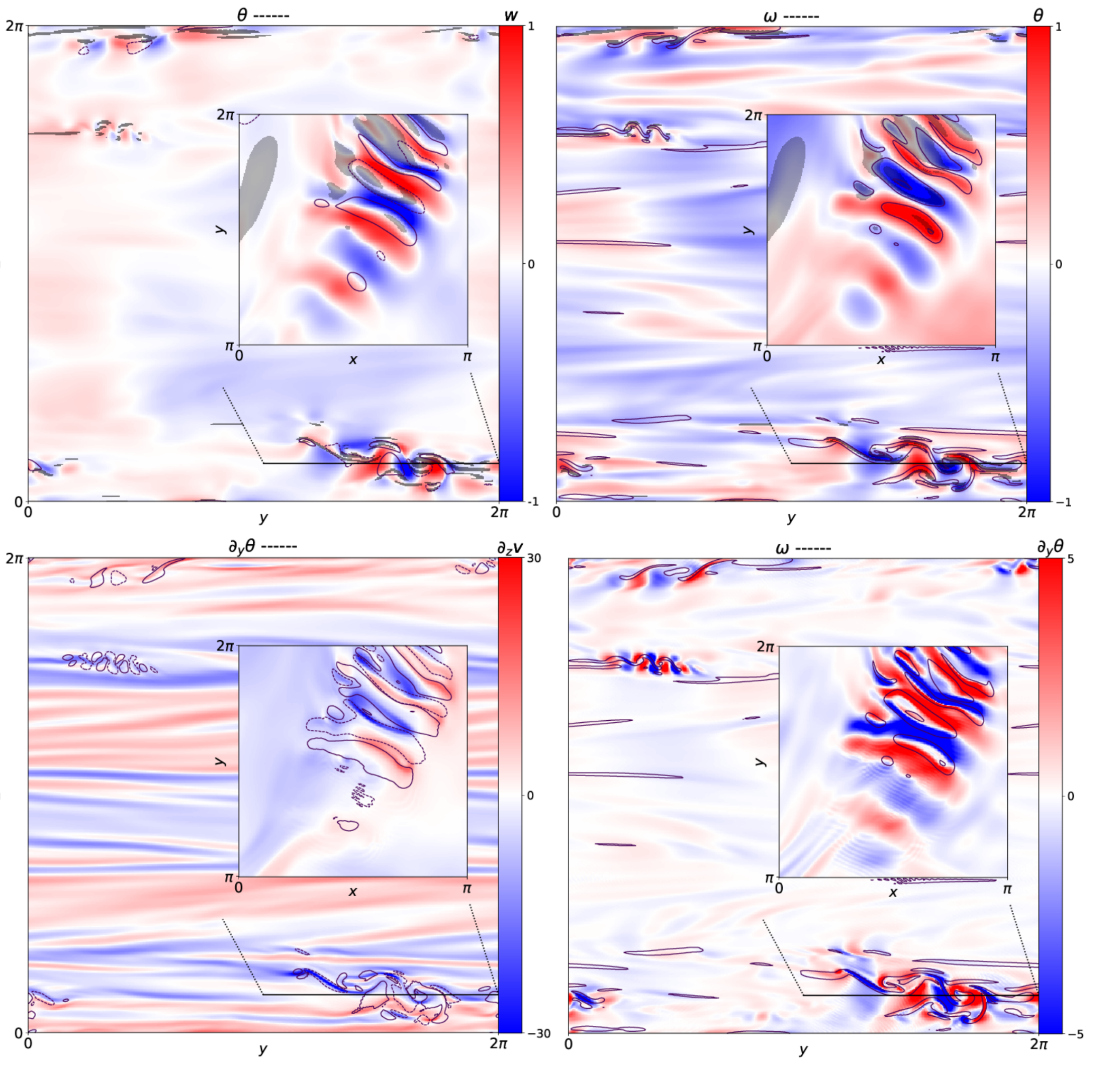
Generation of Large-scale Extreme Drafts by Buoyancy Driven Instabilities

With the use of two-dimensional renderings, we highlighted the correlations between the extreme events in the large- and in the small-scale fields [6]. The figure below show some of these correlations for run *9* (Fr = 0.076, Tab. 1) :



Supported by the observed correlation (see figure) between bursts in the buoyancy field θ and large values of the horizontal gradients ($\partial_z u$ and $\partial_z v$) pointing to the presence of shears, and of vorticity ω, we propose a mechanism for the generation of the extreme vertical drafts developing in the vertical component of the velocity field w [6]. Based on this mechanism, vertically sheared horizontal winds, make the flow prone to develop Kelvin Helmholtz instabilities generating regions where convective motions develop. These motion allow θ to overcome the stable stratification ($\partial_z\theta > N$, gray shaded areas) resulting in the formation of the strong vertical velocity drafts emerging in our analysis. Consistently with this picture, the observed large-scale intermittency vanishes for $Fr < 0.04$, since when N increases (thus Fr decreases) the condition $\partial_z\theta > N$ is hardly satisfied.
The developing convective motions can also amplify the gradients of θ, generating vorticity, which enhancing in turn the local dissipation.

In conclusion, we found that large-scale extreme events in the vertical velocity field are produced by buoyancy driven instabilities, which through the generation of vorticity connect large- and small-scale intermittent dynamics in stratified flows of geophysical interest.